SEQUENCE AND SERIES

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Chapter 1 Introduction

Consider the number 1,2, 3,… this flow of numbers(counting) do follow a definite rule. The next number (term) from the previous is obtained by adding 1.

Consider also …-4, -1, 2, 5, … they are also written one at a time following a certain rule, by adding 3.

The way the two forms of numbers are written is called a ***sequence***, If a sequence is expressed as a sum of we obtain a series:-

i.e 1, 2, 3, 4, …

-4, -1, 2, 5, … these are sequence

And

i.e 1 + 2 + 3 + 4 + 5 + …

-4 + (-1) + 2 + 5 + … these are series

These are two series which follow a finite rule :

1: Arithmetical Progression(AP)

2: Geometrical Progression

ARITHMETICAL PROGRESSION(A.P)

A sequence or series is said to be an A.P , if the difference between any two **succeeding** terms is the same throughout.

Example:

1, 2, 3, 4, …

Or

-4, -1, 2, 5, …

Or

4, 7, 10, …

The terms of an A.P,

We shall denote first term of an A.P by “a” and the common difference by “d”

Then the A.P becomes, a, (a + d), (a + 2d), (a + 3d), …, nth term, …

First term => a + (1-1) d = a + 0*d = a

Second term => a + (2-1) d = a + 1*d = a + d

Third term => a + (3-1) d = a + 2*d = a + 2d

Therefore the nth term => a + (n – 1) d

The general term, A_n (nth term) is given by

$$A_n = a + (n - 1)d$$

Example1:

The 15th term and 17^{th} term of an A.P are 7 and 25 respectively.

Find the 13^{th} term.

Solution:

5^{th} term

a + 4d = 7 …………………(i)

17^{th} term

a + 16d = 25 ……………………..(ii)

by solving the two equations simultaneously,

subtract (i) from (ii)

a + 16d –(a + 4d) = 25 -7

12d = 18

Giving d = 3/2 (common difference)

Substitute the value of d into equation (i),

a + 4*3/2 = 7

a + 6 = 7

giving a = 1

13^{th} term,

Formula A_{13} = a + 12d

Where a = 1 and d = 3/2

Therefore,

A_{13} = a + 12d

= 1 + 12*3/2

$= 1 + 18$

$= 19$

Hence the 13th term is 19

Sum to n terms of an A.P

Let a be 1st term d be common difference and S_n be sum of A.P to n terms

$S_n = a + (a + d) + (a + 2d) + \ldots + (l - d) + l + \ldots$

Where l => $A_n = a + (n-1)d$

The opposite sum also gives the sum Sn

i.e $S_n = l + (l-d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a$

We have,

$S_n = a + (a + d) + (a + 2d) + \ldots + (l - 2d) + (l-d) + l$

$S_n = l + (l-d) + (l-2d) + \ldots + (a + 2d) + (a + d) + a$

$2S_n = (a + l) + (a + l) + (a + l) + \ldots + (a + l) + (a + l)$

There are (a + l), n-brackets

Therefore,

$2S_n = n(a + l)$

Substitute $l = a + (n-1)d$

$S_n = \frac{1}{2}(n)\{a + a + (n - 1)d\}$

$S_n = n/2\{ 2a + (n - 1)d\}$

Example 2:

How many terms of the series 9 + 12 + 15 + … must be taken to make the sum of 306?

Solution:

Series: 9 + 12 + 15 + …

From the series,

a = 9,

d = 3,

Sn = 306

Now from,

Sn = n/2{2a + (n-1)d}

Where,

Sn is the sum of n terms

a is the first term

n is the number of terms

d is the common differnce

306 = n/2{ 2*9 + (n-1)3}

Multiply by 2 both sides,

$612 = 18n + 3n^2 - 3n$

$3n^2 + 15n - 612 = 0$

Divide by 3 throughout giving

$n^2 + 5n - 204 = 0$

solve as quadratic equation by factorization method

sum of factors = 5

product of factors = -204

giving factors 17 and -12

$n^2 + 17n - 12n - 204 = 0$

$n(n + 17) - 12(n + 17) = 0$, the term in brackets must be the same

$(n-12)(n+17) = 0$

Either $n-12 = 0$ or $n+17 = 0$

Giving $n=12$ or $n=-17$

Therefore the number of terms n = 12, since the number of terms must be positive

Arithmetic Mean(A.M)

If three numbers are in an A.P, the middle term is called the arithmetic mean (A.M) between the other two,

If a, b, c are in A.P

$b - a = c - b$

$b + b = a + c$

$2b = a + c$

$b = (a + c)/2$ the A.M

GEOMETRICAL PROGRESSION(G.P)

A sequence of numbers is said to be in G.P if the ratio between any two succeeding numbers(terms) is the same throughout.

Let,

a be the 1st term of G.P

r be the common ratio

Therefore the G.P is $a, ar, ar^2, ar^3, \ldots$

1st term:

$a = ar^{(1-1)} = ar^0$ but $r^0 = 1$

2nd term:

$ar = ar^{(2-1)}$

3rd term:

$ar^2 = ar^{(3-1)}$

.

.

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nth term , $G_n = ar^{(n - 1)}$

The general term (nth term), G_n is given by

$$G_n = ar^{(n - 1)}$$

Example 3

Find the G.P whose fifth term is 48 and 9th term is 768

Solution:

Fifth term:

$G_5 = ar^4 = 48$ ……………………..(i)

9th term:

$G_9 = ar^8 = 768$ ……………………..(ii)

Divide (i) by (ii)

$ar^8 / ar^4 = 768/48$

$r^4 = 16$

$r = +/- 2$

For r =2,

$16a = 48$, giving $a = 3$

Then G.P => 3, 6, 12, 24, 48, …

For r=-2,

Then G.P=> 3, -6, 12, -24, 48,…

SUM OF TERMS OF ANY G.P

The G.P of the 1st term , a and common ratio r is :-

$a, ar, ar^2, ar^3, \ldots , ar^{(n-1)}$

Let S_n be the sum of first n terms of a G.P

$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{(n-2)} + ar^{(n-1)}$

Multiply by r , we obtain

$r S_n = ar + ar^2 + ar^3 + \ldots + ar^{(n-1)} + ar^n$

by subtraction

$S_n - r S_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$$\mathbf{S_n = a(1 - r^n) / (1 - r)}$$

For r < 1

$$\mathbf{S_n = a(r^n - 1) / (r - 1)}$$

For r >1

Example : 4

How many terms of the series 2 + 6 + 18 + … must be taken to get the sum 2186

Solution:

Series : 2+ 6 + 18 + …

Sum = 2186

a = 2

r = 3 (r > 1)

$S_n = a(r^n - 1) / (r - 1)$

For r >1

$2186 = 2(3^n - 1)/(3-1)$

$2186 = 3^n - 1$

$2186 + 1 = 3^n$

$2187 = 3^n$

$3^7 = 3^n$ giving n = 7

Therefore the number of terms must be 7

SUM OF INFINITY OF A G.P

For r < 1

$S_n = a(1 - r^n) / (1 – r)$ if $n \to \infty$

n approaches to infinity

$r^n > 0$

Therefore $S\infty = a/(1-r)$

Example:

Find the sum to infinity of the G.P

$1 + 1/\sqrt{10} + 1/10 + \ldots$

Solution:

$a = 1$

$r<1$

$r = 1/\sqrt{10}$

$S\infty = a/(1-r) = 1/(1 - 1/\sqrt{10})$

$= \sqrt{10} / (\sqrt{10}) - 1$

$= 1.462$

THE GEOMETRICAL MEAN(G.M)

If a, b, c form a G.P the middle term b is called the Geometric mean (G.M) between the other two i.e a and c in a G.P

Suppose a, b, c are in a G.P

Common ratio, $r = b/a = c/b$

Or $b^2 = ac$

Giving $b = \pm\sqrt{(ac)}$

Therefore G.M is given by $b = \pm\sqrt{(ac)}$

Chapter 2 Solved Questions

Q 1 : Find the number of terms in a sequence of numbers between 20 and 200 that are multiple of (a) 6 (b) 11

Solution:

Multiple of 6

24, 30, 36, 42, ……, 192, 198

1^{st} term = 24,

Common difference = 6,

Last term = 198

From $A_n = a + (n-1)d$

Where A_n = last term,

a = first term,

d = common difference

Therefore,

198 = 24 + (n-1)*6

198= 24 + 6n -6

198 + 6 – 24 = 24 – 24 +6n – 6 + 6

198 -24 = 6n

174 = 6n,

Divide by 6 both sides,

174/6 = 6n/6

Therefore number of terms is 29

Multiple of 11

22, 33, 44, ……, 187, 198

1^{st} term = 11,

Common difference = 11,

Last term = 198

From $A_n = a + (n-1)d$

Where A_n = last term,

a = first term,

d = common difference

Therefore,

198 = 22 + (n-1)*11

198= 22 + 11n -11

198 = 11 +11n

198 -11 = 11n

187 = 11n,

Divide by 11 both sides,

187/11= 11n/11

Therefore number of terms is 17

Q 2: The second and the 16^{th} terms of an A.P are 76 and 20 respectively. Find the first term and the common difference

Solution:

2^{nd} term = 76,

16^{th} term = 20,

Required : first term and common difference

From An = a + (n-1)d

Where

A_n = last term,

a = first term,

d = common difference

A2 = a + (2-1)d = 76

a + d = 76 ………………………..(i)

A16 = a + (16-1)d = 20

a + 15d = 20 ……………………….(ii)

Solve (i) and (ii) simultaneously by subtracting (i) from (ii)

a – a + 15d – d = 20 -76

14d = -56 giving d = -4

Substitute the value of d to equation (i)

a + (-4) = 76

a – 4 = 76

add 4 to both sides,

a – 4 + 4 = 76 + 4

a = 80

Hence the first term is 80 and the common difference is -4

Q 3: The third and the ninth terms of an A.P are 4 and 21 respectively. Find the first term and the common difference

Solution:

3^{rd} term = a + (3-1)d = 4

a + 2d = 4 …………………(i)

9^{th} term = a + (9-1)d = 21

a + 8d = 21 ……………….(ii)

subtract (i) from (ii)

a – a + 8d – 2d = 21 – 4

6d = 17 giving d = 17/6

Substitute the value of d into equation (i)

a + 2*17/6 = 4

a + 17/3 = 4

subtract 17/3 both sides

a = 4 – 17/3 = (12-17)/3 = -5/3

Hence the common difference is 17/6 and the first term is -5/3

Q 4: The 5th term of a geometric progression is 18 and 8th term is 144. Find the first term and the common ration.

Solution :

5th term of G.P = 18

$G_5 = ar^{(5 - 1)} = 18$

$ar^4 = 18$ ……………………………..(i)

where,

a = first term

r = common ration

$G_8 = ar^{(8 - 1)} = 144$

$ar^7 = 144$ ……………………………..(ii)

divide (ii) by (i)

$ar^7 / ar^4 = 144/18$

$r^3 = 72/9 = 8$

giving r = 2

substitute the value of r into equation (i)

$a*2^4 = 18$

$a * 16 = 18$

by dividing 16 both sides a = 18/16 = 9/8

Hence the value of a = 9/8 and r = 2

Q 5: The 3rd and 5th terms of a geometric progression are 104 and 26 respectively. Given that the common ratio is negative, find

a) The first term

b) The common ratio

c) The 10th term

Solution:

3rd term = 104

$G3 = ar^{(3-1)} = 104$

$ar^2 = 104$ ……………………………(i)

5th term = 26

$G5 = ar^{(5-1)} = 26$

$ar^4 = 26$ ……………………………(ii)

divide (ii) by (i)

$ar^4 / ar^2 = 26/104$

$r^2 = ¼$ giving r = +/- ½

for r = +1/2

using (i)

$ar^2 = 104$, but $r^2 = ¼$

$a * ¼ = 104$

multiply by 4 both sides,

$a = 416$

and for $r = -1/2$

using (i) the value of a will be the same

the 10th term

$G_{10} = ar^{(10-1)} = 416 * ½^{(9)} = 416/2^9 = 416/512 = 52/64 = 27/32$

Hence the first term is 416, the common ration is +/- ½ and the 10th term is 27/32

Q 6: Find the sum of 15 terms of the series 11 + 16 + 21 + …

Solution:

The series is in A.P

The first term = 11,

Common difference = 5,

From $Sn = n/2\{ 2a + (n-1)d\}$

Where,

Sn = sum of terms,

n = number of terms,

a = first term

d = common difference

therefore Sn = 15/2{2*11 + (15-1)5}

Sn = 15/2{22 + 70}

Sn = 15/2 * 92

Sn = 15*46 = 690

Hence the sum of 15 terms is 690

Q 7 : Find the sum of all integers between 1 and 100 that are divisible by 6

Solution :

Series = 6, 12, 18, 24, 30, …. 92, 96

First term = 6,

Common difference = 6,

Sum Sn = n/2{2a + (n-1)d}

First we find the number of terms, n

From An = a + (n – 1)d

Where An = last term,

n = number of terms,

d = common difference

96 = 6 + (n -1)6

96 = 6 + 6n -6

96 = 6n giving n = 16 by dividing by 6 to both sides

Therefore ,

S16 = 16/2{2*6 + (16-1)6}

S16 = 8{12 + 90)

S16 = 8*112 = 896

Hence the sum of terms is 896

Q 8: Find the sum of all the integers between 0 and 200 that are divible by 11

Solution:

Series : 11, 22, 33, 44, … , 187,198

1^{st} term = 11

Common difference = 11,

Required : Sum of terms

Number of terms:

An = a + (n – 1)d

198 = 11 + (n – 1) * 11

198 = 11 + 11n – 11

198 = 11n

Giving number of terms n = 18

Sn = n/2{2a + (n-1)d}

Sn = 18/2{2 * 11 + (18-1)11}

Sn = 9{22 + 187}

Sn = 9*209

Sn = 1881

Hence the sum of terms is 1881

Q 9 : How many terms are in the A.P 4, 10, 16, … add up to 310?

Solution:

4, 10, 16, …

a = 4

d = 6

sum = 310

from,

Sn = n/2{2a + (n-1)d}

310 = n/2{2*4 + (n-1)6}

310 = n/2{8 + (n-1)6}

310 = n/2{8 + 6n – 6}

310 = n/2(6n + 2)

$310 = 3n^2 + n$

Rearranging the equation to have quadratic equation

$3n^2 + n - 310 = 0$

Solve as quadratic equation

By factorization method,

Sum of factors = +1

Product of factors = -930

2	930
3	465
5	155
31	31
	1

Prime factors 2, 3, 5, 31

2*3*5*31 = 930

30*31 = 930

Therefore factors are +31 and -30

$3n^2 - 30n + 31n - 310 = 0$

$3n(n - 10) + 31(n - 10) = 0$

$(3n + 31)(n - 10) = 0$

Either $3n + 31 = 0$

$3n = -31$

Giving $n = -31/3$

Or $n-10 = 0$

Giving n = 10, number of terms should be positive

Therefore number of terms is 10

Q 10: The first three terms of an A.P are x – 1 , 2x and 5x -2 . Find the value of x and the sum of the first 8 terms

Solution:

1^{st} term = x -1

2^{nd} term = 2x

3^{rd} term = 5x -2

2^{nd} term – 1^{st} term = 3^{rd} term – 2^{nd} term = Arithmetic Mean

2x – (x-1) = 5x -2 –2x

2x -x + 1 = 5x -2x -2

x + 1 = 3x -2

giving x = 3/2

therefore;

1^{st} term = 1/2

2^{nd} term = 3

3^{rd} term = 11/2

Thus, d = 5/2

Sn = n/2{2a + (n-1)d}

Where

n = 8,

d = 5/2

a = 1/2

Sn = 8/2{2*1/2 + (8 – 1)5/2}

Sn = 4{1 + 35/2}

Sn = 4 + 70

Sn = 74

Hence the value of x is 3/2 and sum of 8 terms is 74

Q 11: If x + 4 , 2x – 2 and x are three consecutive terms of an A.P, find the value of x

Solution :

x + 4 , 2x – 2, x

2x -2 –(x +4) = x – (2x -2)

2x – 2 -x – 4 = x -2x + 2

x -6 = -x + 2

x + x = 2 + 6

2x = 8

Giving the value of x = 4

Q 12: Find the sum of 9 terms of the following series ½ + ¼ + 1/8 + …

Solution:

½ + ¼ + 1/8 + …

Series is a G.P

1st term = ½,

Common ratio = ½

Number of terms n = 9

Sum $S_n = a(1 - r^n) / (1 - r)$ for $r < 1$

$Sn = ½\{1 - (1/2)^9\} / (1 - 1/2)$

$S9 = ½\{1 - (1/2)^9 / (1/2)$ complete as part of exercise

Q 13: Find the sum of 6 terms of the following series 128 + 64 + 32 + …

Solution :

128 + 64 + 32 + …

The series is in a G.P

1st term = 128

Common ratio = 1/2

Sum $S_n = a(1 - r^n) / (1 - r)$ for $r < 1$

$S_6 = 128(1 - (1/2)^6) / (1 - 1/2)$

$S_6 = 128(1 - (1/2)^6) / (1/2)$ complete as part of exercise

Q 14: Find the number of terms of the following series 1/27 + 1/9 + 1/3 + … + 243

Solution:

1/27 + 1/9 + 1/3 + … + 243

The series is in a G.P

1^{st} term = 1/27

Common ration = 3

From

$G_n = ar^{(n - 1)}$

$Gn = 1/27*3^{(n-1)} = 243$

Multiply by 27 both sides,

$3^{(n-1)} = 243*27$

3	243
3	81
3	27
3	9
3	3
	1

3	27
3	9
3	3
	1

$3^{(n-1)} = 3^5*3^3$

$3^{(n-1)} = 3^8$

since the bases are equal ,the exponents should be also equal giving n-1 = 8 , n = 9

Therefore the number of terms is 9

Q 15 : The 4th and 7th terms of a G.P are 144 and 18 respectively.

Find :

a) The common ration

b) The first term

c) The sum of the first six terms

Solution:

4th term = 144

$G_4 = ar^{(n-1)} = 144$

$ar^{(4-1)} = 144$

$ar^3 = 144$ ……………………….(i)

7th term = 18

$G_7 = ar^{(n-1)} = 18$

$ar^{(7-1)} = 18$

$ar^6 = 18$ ………………………….(ii)

divide (ii) by (i)

$ar^6 / ar^3 = 18/144$

$r^3 = 1/8$

$r^3 = (\frac{1}{2})^3$

giving $r = \frac{1}{2}$

using (i)

$ar^3 = 144$ but $r^3 = 1/8$

a * 1/8 = 144

Multiply by 8 both sides,

a = 144*8 = 1152

Sum of first 6 terms:

Sum $S_n = a(1 - r^n) / (1 – r)$ for $r < 1$

$S_n = 1152\{1 – (1/2)^6\} / (1 – 1/2)$ complete as part of exercise

Q 16: In a G.P , the 5^{th} and 7^{th} terms are -2 and -18 respectively.

a) Find the possible values of the common ratio

b) If the common ratio is positive find the sum of the first 6 terms of the series

Solution:

5^{th} term = $G_5 = ar^{(n - 1)} = -2$

$ar^{(5 - 1)} = -2$

$ar^4 = -2$ ……………………….(i)

7^{th} term = $G_7 = ar^{(n-1)} = -18$

$ar^{(7-1)} = -18$

$ar^6 = -18$ ……………………….(ii)

divide (ii) by (i)

$ar^6 / ar^4 = -18/-2$

$r^2 = 9$

r = +3 or r = -3

Therefore possible values of common ratio is +3 or -3

For positive common ratio, r = +3

Substitute into (i)

$ar^4 = -2$

$a\,3^4 = -2$

a * 81 = -2

a = -2/18 = -1/9

Sum of 6 terms

$S_n = a(r^n - 1) / (r - 1)$

For r >1

$S_n = a(r^n - 1) / (r - 1)$

$S6 = -1/9 * (3^6 - 1) / (3 - 1)$

$S6 = -1/9 * (3^6 - 1) / 2$

$S6 = -1/9 * 728/2$

$S6 = -81/2 = -40.5$

The sum of the first 6 terms of the series = -40.5

Q 17: If 8 + x + 18 is a G.P, find;

a) The possible values of x

b) The sum of the first 5 terms if x is the second term and it is positive

Solution ;

8 + x + 18

Possible values of x

x/8 = 18/x

Cross multiplication

$x^2 = 18 * 8 = 144$

x = +12 or x = -12

8 + 12 + 18

Common ratio 12/8 = 3/2

Sum of 5 terms

$S_5 = 8\{(3/2)^5 – 1\} / (12- 1)$

$S_5 = 8\{(3/2)^5 – 1\} / (11)$ complete as part of exercise

Q 18 : Write down the first three terms and 15^{th} term of the sequence whose nth term is n/(n+1)

Solution:

n/(n+1)

for n=1, first term =

for n= 2, second term =

for n= 3, 3^{rd} term =

for n=4 , fourth term =

Q 19: Find the 20^{th} term of the series 8 – 4 + 2 – 1 + …

Solution:

8 – 4 + 2 – 1 + …

The series is in a G.P

Common ratio r = -4/8 = 2/-4 = -1/2

First term = 8

From Gn = $ar^{(n - 1)}$

$G_{20} = 8(-1/2)^{(20 - 1)}$ complete as part of your exercise

Q 20 : Find the sum of 10 terms of 1/3 + 1/9 + 1/27 + …

Solution:

1/3 + 1/9 + 1/27 + …

The series is in G.P

Where common ratio, r = 1/3

First term = 1/3

Sum $S_n = a(1 - r^n) / (1 - r)$ for $r < 1$

$S_{13} = 1/3(1 - (1/3)^{13}) / (1 - 1/3)$ for $r < 1$ complete as part of your exercise

Q 21: Find the sum of 13 terms of 3 + 6 + 9 + 12 + …

Solution:

3 + 6 + 9 + 12 + …

The series is in a A.P

Common difference = 3

First term = 3

Sn = n/2{2a + (n – 1)d}

S13 = 13/2{2*3 + (13 – 1)*3}

S13 = 13/2{6 + 12*3}

S13 = 13/2*42

S13 = 13*21 = 273

Hence the sum of 13 terms is 273

Q 22: Find 7^{th} term and 18^{th} term of an arithmetic series are 6 and 22 ¼ respectively. Calculate the sum of :

a) Common difference

b) First term

c) Value of n if the sum of the first n terms of the series is 252

Solution:

7^{th} term = a + 6d = 6 ……………………………(i)

18^{th} term = a + 17d = 22 ¼ …………………..(ii)

Solve (i) and (ii) simultaneously by subtracting (i) from (ii)

a – a + 17d - 6d = 22 ¼ - 6

11d = 16 ¼

11d = 65/4

Divide by 11 both sides

d = 65/(4*11) = 65/44

common difference = 65/44

substitute the value of d into (i)

a + 6*65/44 = 6

a + 390/44 = 6

a = 6 – 390/44

a = (44*6 – 390)/44 = (264 – 390)/44 = -126/44 = -63/22

The first term is -63/22

Complete part (c)

Q 23: Find the sum of all integers between 0 and 200 that are multiple of 6

Solution:

6,12,18,24,30,….., 192, 198

First term = 6

Common difference = 6

Last term = 198

From $A_n = a + (n - 1)d$

$198 = 6 + (n-1)6$

$198 = 6 + 6n - 6$

$198 = 6n$. giving $n = 33$

From $S_n = n/2\{2a + (n - 1)d\}$

$S_{33} = 33/2\{2*6 + (33 - 1)*6\}$

$S_{33} = 33/2\{12 + 32*6\}$

$S_{33} = 33/2\{12 + 192\}$

$S_{33} = 33/2(204)$

$S_{33} = 33*102$

$S_{33} = 3366$

Therefore the sum of all integers between 0 and 200 is 3366

Q 24: The 3rd and 6th terms of a geometric series are 27 and 8 respectively. Find the common ratio and the 9th term

Solution :

From $G_n = ar^{(n - 1)}$

3th term = $G_3 = ar^{(3 - 1)}$

$G_3 = ar^{(3 - 1)} = ar^2 = 27$

$ar^2 = 27$ ……………………………(i)

6th term = $G_6 = ar^{(6-1)}$

$G_6 = ar^{(6-1)} = ar^5 = 8$

$ar^5 = 8$ ……………………………….(ii)

divide (ii) by (i)

$ar^5 / ar^2 = 8/27$

$r^3 = 2^3 / 3^3$

r = 2/3,

common ratio = 2/3

substitute the value of r ino (i)

$a(2/3)^2 = 27$

a*4/9 = 27

a = 27*9/4 = 243/4

9th term:

From $G_n = ar^{(n-1)}$

$G_9 = ar^{(9-1)} = 243/4 * (2/3)^8 = 243/4 * 16*16/81*81 = 64/27$

Therefore the 9th term is 64/27

Q 25 : If 2, x + 1 and 18 are three consecutive terms of a G.P, What are the possible values of x?

Solution:

2, x +1 and 18 is a G.P

Therefore (x + 1)/2 = 18/(x + 1)

Cross multiplication

$(x + 1)(x + 1) = 2 * 18$

$x^2 + x + x + 1 = 36$

$x^2 + 2x + 1 = 36$

$x^2 + 2x - 35 = 0$

Sum of factors = +2

Product of factors = -35

Factors are +7 and -5

$x^2 + 7x – 5x - 35 = 0$

$x(x + 7) – 5(x + 7) = 0$

$(x – 5)(x + 7) = 0$

Either $x - 5 = 0$ or $x + 7 = 0$

Giving $x = 5$ or -7

Therefore the possible values of x are 5 or -7

Q 26: Find three numbers in arithmetical progression such that their sum is 27 and their product is 504

Solution:

$a + (a + d) + (a + 2d) = 27$

$3a + 3d = 27$

$a + d = 9$ ………………………..(i)

$a *(a + d)* (a + 2d) = 504$

$(a^2 + ad)(a + 2d) = 504$

$a^2 *a + a^2 *2d + ad*a + ad*2d = 504$

$a^3 + 2da^2 + da^2 + 2d^2a = 504$ ……………………..(ii)

From (i) , $d = 9 - a$ ………………………………..(iii)

Substitute (iii) into (ii)

$a^3 + 2(9 - a)a^2 + (9 - a)a^2 + 2(9 - a)^2a = 504$

$a^3 + 18a^2 - 2a^3 + 9a^2 - a^3 + 2a(81 - 18a + a2) = 504$

$a^3 + 18a^2 - 2a^3 + 9a^2 - a^3 + 162a - 36a^2 + 4a^2 = 504$

$a^3 - 2a^3 - a^3 + 18a^2 + 9a^2 - 36a^2 + 4a^2 + 162a = 504$

$-2a^3 - 5a^2 + 162a - 504 = 0$ to be finished as part of exercise

Q 27: Insert seven arithmetic means between 2 and 26

Solution:

Arithmetic mean of a, b, c

$b - a = c - b$

$b + b = a + c$

$2b = a + c$

$b = (a + c)/2$

2, 5, 8, 11, 14, 17, 20, 23, 26

1st arithmetic mean

$(2 + 26)/2 = 28/2 = 14$

2, (14), 26

2nd arithmetic mean

$(2 + 14)/2 = 16/2 = 8$

2, (8), 14, 26

3^{rd} arithmetic mean

(2 + 8)/2 = 10/2 = 5

2, (5), 8, 14, 26

4^{th} arithmetic mean

(8 + 14)/2 = 22/2 = 11

2, 5, 8, (11), 14, 26

5^{th} arithmetic mean

(14+ 26)/2 = 40/2 = 20

2, 5, 8, 11, 14, (20), 26

6^{th} arithmetic mean

(14+ 20)/2 = 34/2 = 17

2, 5, 8, 11, 14, (17), 20, 26

7^{th} arithmetic mean

(20+ 26)/2 = 46/2 = 23

2, 5, 8, 11, 14, 17, 20, (23), 26

Q 28: The first term of an arithmetical progression is 25 and the third term is 19. Find the number of terms in the progression if its sum is 82

Solution:

1^{st} term = 25

3^{rd} term = a + 2d = 19

$25 + 2d = 19$

$2d = 19 - 25$

$2d = -6$

$d = -3$

Sum of terms:

$S_n = n/2\{2a + (n - 1)d\}$

$82 = n/2\{2*25 + (n - 1)*-3\}$

$82 = n/2\{50 + 3 - 3n\}$

Multiply by 2 both sides

$82*2 = n(53 - 3n)$

$164 = 53n - 3n^2$

$3n^2 - 53n + 164 = 0$

Solve quadratic equation using factorization method

Sum of factors = -53

Product of factors = +492

2	492
2	246
3	123
41	41
	1

Factors are -12 and - 41

$3n^2 - 12n - 41n + 164 = 0$

$3n(n - 4) - 41(n - 41) = 0$

$(3n - 41)(n - 4) = 0$

Either $3n - 41 = 0$ or $n - 4 = 0$

Giving $n = 41/3$ or $n = 4$

Therefore the number of terms = 4

Q 29 : Insert three geometric means between 162 and 1250

Solution :

From geometric mean formula

$b^2 = \sqrt{ac}$

162, 270, 450, 750, 1250

1st term insertion

162, (450), 1250

1st geometric mean = $\sqrt{(162*1250)} = \sqrt{202500} = 450$

2nd term insertion

162, (270), 450, 1250

2nd geometric mean = $\sqrt{(162*450)} = \sqrt{72900} = 270$

3rd term insertion

162, 270, 450, (750) 1250

3rd geometric mean = $\sqrt{(450*1250)} = \sqrt{562500} = 750$

Q 30 : Find the sum of ten terms of the geometrical series 2, -4, 8, …

Solution:

2, -4, 8, …

1st term = 2,

Common ration = -4/2 = -2

Number of terms = 10

From sum formula,

$S_n = a(1 - r^n) / (1 – r)$ for $r < 1$

$S_{10} = 2\{1 – (-2)^{10}\} / \{1 – (-2)\}$

$S_{10} = 2(1 – (-2)^{10}) / (1 + 2)$

$S_{10} = 2(1 – 1024) / (1 + 2)$

$S_{10} = 2(– 1023) / (3)$

$S_{10} = – 2046/ 3 = -682$

Therefore the sum of ten terms = -682

Q 31: Write down the first three terms and the 8th term of the series whose nth term is 4n – 5

Solution:

nth term = 4n – 5 and the formula for nth term is a + (n-1)d

therefore a + (n-1)d = 4n – 5

for n=1,

a + (n-1)d = 4n -5

a + (1-1)d = 4*1 -5

a + 0 = 4-5

a = -1, therefore the first number is -1

for n=2,

a + (n-1)d = 4n -5

a + (2-1)d = 4*2 -5

a + d = 8-5

a + d = 3, but a = -1

-1 +d = 3

d = 3 +1 = 4, therefore the common difference is 4

The first terms = -1,

The second term = -1 +4 = 3

The third term = 3 +4 = 7

Therefore the first three terms is -1,3,7

8^{th} term,

From the formula of nth term, a + (n-1)d, where d = 4, a = -1

8^{th} term = -1 +(8-1)*4 = -1 +7*4 = -1 + 28 = 27

Therefore the 8^{th} term is 27

Q 32 : Find the sum of ten terms of an arithmetical progression of which the first term is 60 and the last is -104

Solution :

Number of terms = 10

1st term = 60

Last term = -104

From An = a + (n-1)d

Where,

An = last term

a = first term

n = number of terms

d = common difference

therefore ,

-104 = 60 + 9d

-104 – 60 = 9d

-164 = 9d, giving d = -164/9

From the sum formula

Sn = n/2{2a + (n-1)d}

S_{10} = 10/2{2*60 + (10-1)*-164/9

S_{10} = 5{120 + 9*-164/9}

S_{10} = 5(120 -164)

S_{10} = -220

Therefore the sum of ten terms = -220

Q 33: If the first, third and sixth term of an arithmetical progression are in geometrical progression, find the common ration of the geometrical progression.

Solution:

Terms of A.P

1^{st} term of A.P = a

3^{rd} term of A.P = a + 2d

6^{th} term of A.P = a + 5d

Terms of G.P

(a + 2d)/a = (a + 5d)/ (a + 2d) = common ratio

By crossing multiplication

(a + 2d)(a + 2d) = (a +5d)a

a*a + a*2d + 2d*a + 2d*2d = a*a +a*5d

$a^2 + 2da + 2da + 4d^2 = a^2 + 5da$

$4da + 4d^2 = 5da$

$4d^2 = 5da - 4da$

$4d^2 = da$

Divide by d both sides,

4d = a ……………………………..(i)

Substitute (i) into A.P terms to find the common ratio

(a + 2d)/a = (a + 5d)/ (a + 2d) = common ratio

(4d + 2d) /4d = common ratio

6d/4d = common ratio

3/2 = common ratio

Therefore the common ratio = 3/2

Q 34 : The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, find the last term.

Solution:

a = ?

r = ?

n = ?

last term = ?

From the nth term of G.P

$G_n = ar^{(n - 1)}$

The last three terms:

Last term = $G_n = ar^{(n - 1)}$

Last but one = $G_{n-1} = ar^{(n - 2)}$

Last but two = $G_{n-2} = ar^{(n - 3)}$

The first three terms:

1^{st} term= $G_1 = a$

2^{nd} term = $G_2 = ar^{(2 - 1)} = ar$

3^{rd} term = $G_3 = ar^{(3 - 1)} = ar^2$

But 3rd term = 5

Therefore,

$ar^2 = 5$ ……………………..(i)

$ar^{(n-1)} + ar^{(n-2)} + ar^{(n-3)} = 1024\{a + ar + ar^2\}$

$ar^{(n-1)} + ar^{(n-2)} + ar^{(n-3)} = 1024a + 1024ar + 1024ar^2$

divide a throughout

$r^{(n-1)} + r^{(n-2)} + r^{(n-3)} = 1024 + 1024r + 1024r^2$ to be finished as part of exercise

Q 35: Find two numbers whose arithmetic mean is 39 and geometric mean is 15

Solution:

A.M = 39

Arithmetic Mean

a, b, c

$b = (a + c)/2 = 39$

$a + c = 78$ ………………….(i)

G.M = 15

Geometric Mean

a, b, c

$b = \sqrt{(ac)} = 15$

$\sqrt{(ac)} = 15$

$ac = (15)^2 = 225$ ……………………..(ii)

from (i) $a = 78 - c$

Substitute $a = 78 - c$ into (ii)

$c(78 - c) = 225$

$78c - c^2 = 225$

$c^2 - 78c + 225 = 0$

Solve quadratic equation

Sum of factors = -78

Product of factors = 225

Complete this question

Q 36: The second and the third terms of a geometrical progression are 24 and 12(b + 1) respectively. Find b if the sum of the first three terms of the progression is 76.

Solution:

From $G_n = ar^{(n-1)}$

$G2 = ar^{(2-1)}$

$G_2 = ar = 24$ ……………………(i)

$G3 = ar^{(3-1)}$

$G_3 = ar^2 = 12(b + 1)$ ……………………………(ii)

Divide (ii) to (i)

$ar^2/ar = 12(b + 1) / 24$

$r = 12(b + 1) / 24$ …………………………………(iii)

from $S_n = a(r^n - 1) / (r - 1)$

For r >1

$S_3 = a(r^3 - 1) / (r - 1) = 76$

$a(r^3 - 1) / (r - 1) = 76$ ………………………….(iv) complete as part of exercise

Q 37: Find the sum of the numbers divisible by 3 which lies between 1 and 100. Find also the sum of the numbers from 1 to 100 inclusive which are not divisible by 3.

Solution:

Numbers divisible by 3,

3,6,9,12,15,……90,93,99

The series is A.P

Common difference is 3

First term is 3

Number of terms is determined by using the following formula

$A_n = a + (n – 1)d$

$99 = 3 + (n - 1)3$

$99 = 3 + 3n – 3$

$99 = 3n$

Giving n = 33

Sum is determined by using the following formula

$S_n = n/2\{ 2a + (n – 1)d\}$

S_{33} = 33/2{ 2*3 + (33 – 1)3}

S_{33} = 33/2{ 6 + 32*3}

S_{33} = 33/2{ 6 + 96}

S_{33} = 33/2{ 102}

S_{33} = 33*51

S_{33} = 33*51 = 1683

Therefore the sum is 1683

Numbers which is not divisible by 3,

1,2,4,5,7,8,10,11,…….97,98,100

Since the number of terms which is divisible by 3 is 33, therefore the number of terms which not divisible by 3 is 100 – 33 = 67

Q 38: Find how many terms of the progression 5 + 9 + 13 + 17 + ….. have a sum of 2414

Solution:

Given

Series 5 + 9 + 13 + 17 + …..

Where.

First term is 5,

Common difference is 4

Sum is 2414

From S_n = n/2{ 2a + (n – 1)d}

Where,

Sn = 2414,

a = 5,

d = 4,

needed =number of terms

2414 = n/2{ 2*5+ (n – 1)4}

2414 = n/2{ 10 + 4n – 4}

2414 = n/2{ 6 + 4n }

2414 = n*3 + n *2n

$2n^2 + 3n = 2414$

$2n^2 + 3n - 2414 = 0$

Solve as quadratic equation,

Q 39 : The first term of an A.P is 3. Find the common difference if the sum of the first 8 terms is twice the sum of the first 5 terms.

Solution;

1st term = 3,

Sum of 8 terms is equal to twice sum of 5 terms

From S_n = n/2{ 2a + (n – 1)d}

S_8 = 8/2{ 2*3 + (8 – 1)d}

S_5 = 5/2{ 2*3 + (5 – 1)d}

8/2{ 2*3 + (8 – 1)d} = 2*5/2{ 2*3 + (5 – 1)d}

4(6 + 7d) = 5(6 + 4d)

24 + 28d = 30 + 20d

$28d - 20d = 30 - 24$

$8d = 6$, giving the common difference, $d = 4/3$

Q 40: The fifth term of an arithmetical progression is 24 and the sum of the first five terms is 80. Find the first term, the common difference and the sum of the first fifteen terms of the progression.

Solution:

From

$A_n = a + (n-1)d$,

$A_5 = a + (4-1)d = 24$

$a + 3d = 24$……………………………….(i)

From $S_n = n/2\{ 2a + (n - 1)d\}$

$S_5 = 5/2\{ 2a + (5 - 1)d\} = 80$

$5/2\{ 2a + 4d\} = 80$

Multiply by 2 both sides,

$5\{ 2a + 4d\} = 80*2$

Divide by 5 to both sides,

$\{ 2a + 4d\} = 80*2/5$

$2a + 4d = 32$……………………………………..(ii)

Solve (i) and (ii) simultaneously,

From (i) $a = 24 - 3d$……………………..(iii)

Put (iii) into (ii)

$2(24 - 3d) + 4d = 32$

$48 - 6d + 4d = 32$

48 – 2d = 32

48 – 32 = 2d

16 = 2d , giving d = 8

Substitute the value of d = 8 into (iii) to obtain the value of a

a = 24 – 3*8 = 0

the first term = 0

sum of fifteen terms

S_{15} = 15/2{ 2*0 + (15 – 1)8}

S_{15} = 15/2{ 0 + 14*8}

S_{15} = 15/2{ 112}

S_{15} = 15/2{ 112}

S_{15} = 15*56

S_{15} = 840

Therefore the sum of fifteen terms is 840

Q 41: The sum to n terms a certain A.P is 2n(n + 5). What is (i) its nth term (ii) its constant difference

Solution:

Sum = 2n(n + 5).

From S_n = n/2{ 2a + (n – 1)d}

Sum of first term = 2*1(1+ 5), by putting n = 1

Sum of first term = 2*1(1+ 5) = 2*6 = 12

Sum of first 2 terms = 2*2(2+ 5), by putting n = 2

Sum of first term = 2*2(2+ 5) = 4*7 = 28

Sum of first 3 terms = 2*3(3+ 5), by putting n = 3

Sum of first 3 terms = 2*3(3+ 5) = 6*8 = 48

Therefore,

the 1st term = 12

the 2nd term = 28 – 12 = 16

the 3rd term = 48 – 28 = 20

the sequence is 12, 16, 20…..

first term = 12,

nth term,

An = a + (n – 1)d,

= 12 + (n – 1)4

= 12 + 4n – 4

Therefore the nth term is:

$A_n = 8 + 4n$

common difference = 4

Q 42 : Find the last term and the sum of 7 terms of the following G.P 2 + 6 + 18 + …..

Solution:

2 + 6 + 18 + …..

1st term = 2,

Common ratio = 3

Last term,

From the following formula

$G_n = ar^{(n - 1)}$

where,

1st term = 2,

Common ratio = 3

$G_n = 2*3^{(n - 1)}$

therefore the last term is $2*3^{(n - 1)}$

the sum of 7 terms

from $S_n = a(r^n - 1) / (r - 1)$

For r >1

$S_7 = 2(3^7 - 1) / (3 - 1)$

$S_7 = 2(3^7 - 1) / 2$

$S_7 = (3^7 - 1) = 27*81 – 1 = 2187 – 1 = 2186$

Therefore the sum of 7 terms = 2186

Q 43 : Calculate the sum of to infinity of the following G.P

$9 - 6 + 4 - \ldots$

Solution:

$9 - 6 + 4 - \ldots$

From the following formula

$S_\infty = a/(1-r)$

$a = 9,$

$r = -2/3$

$S_\infty = 9/\{1-(-2/3)\}$

$S_\infty = 9/(1 + 2/3)$

$S_\infty = 9/(5/3)$

$S_\infty = 9 * 3/5 = 27/5$

Q 44 : The sum to infinity of a geometric progression is five times its first term. Find its common ratio, can the sum to infinity ever be 2/3 of the first term? Can it be 1/3 of the first term?

Solution:

From the following formula

$S_\infty = a/(1-r)$, sum to infinity

$a/(1-r) = 5*a$

multiply by $(1 - r)$ to both sides,

$a = 5a(1 - r)$

$1 = 5(1 - r)$

$1/5 = 1 - r$

$r = 1 - 1/5$

$r = 4/5$

common ratio = $4/5$

$S_\infty = a/(1- 4/5) = a/(1/5) = 5a$

Resources

For more educational materials especially questions and answers please visit Loyal Academy, this is the Q&A Platform for school subjects from Primary Schools to University

- Helpful websites: https://loyalacademy.co.tz

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