MATRICES

GEMWAGA NDEGE

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Chapter 1 Introduction

Matrices

A matrix is a rectangular array of information (numbers or letters) arranged in rows and columns. In matrix the numbers or letters enclosed in brackets are called elements, and the brackets either curved or squared. Matrices are named by using capital letters. Elements of a matrix is separated by space, we are not using comma to separate the elements.

Matrix order.

The order of a matrix is the number of rows and the number columns it has. Suppose the matrix has the rows ‘m’ and the columns ‘n’, the matrix is said to be m x n, and we read as “m by n matrix”. Always the number of rows comes first followed by the number of columns

Types of Matrix

A **row matrix** has only one row of elements

(1) 1 x 1 matrix

$\begin{pmatrix} 1 & -2 \end{pmatrix}$ 1 x 2 matrix

$\begin{pmatrix} 1 & -2 & 3 \end{pmatrix}$ 1 x 3 matrix

A **column** matrix has only column of elements

(4) its order is 1 x 1 matrix

$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ its order is 2 x 1 matrix

$\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$ its order is 3 x 1 matrix

A **square matrix** has equal number of or rows and columns, whose order is of the form, m x m.

(6) its order is 1 x 1 matrix

$\begin{pmatrix} 3 & -4 \\ 1 & 7 \end{pmatrix}$ its order is 2 x 2 matrix

$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$ its order is 3 x 3 matrix

Equal matrix the matrix of the same number of rows and the number of columns and their corresponding elements are also equal

Example: 1

$$\begin{pmatrix} 3 & -4 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & 7 \end{pmatrix}$$

A Null or Zero matrix is the matrix that has all elements equal to zero

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Addition of Matrices

Matrices are added if they are of the same order by adding the corresponding elements

M= $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

N = $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

Add A and B

M + N = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ + $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

= $\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$

Subtraction of Matrices

Matrices are subtracted if they are of the same order by subtraction of the corresponding elements

M= $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

N = $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

Subtract A and B

M + N = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ - $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

= $\begin{pmatrix} a- e & b-f \\ c-g & d-h \end{pmatrix}$

Multiplication of Matrices

Multiplication of a matrix by a scalar

Example: 2

if A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find 4A

4A = $\begin{pmatrix} ax4 & bx4 \\ cx4 & dx4 \end{pmatrix}$

= $\begin{pmatrix} 4a & 4b \\ 4c & 4d \end{pmatrix}$

Multiplication of a matrix by a matrix, this is possible only when the number of columns is equal to the number of rows of the second matrix.

Example: 3

A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

B = $\begin{pmatrix} e \\ f \end{pmatrix}$

Since the number of columns of the first matrix = 2

And

The number of rows of the second matrix is 2,

Then multiplication AxB is possible,

Therefore AxB = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} axe + bxf \\ cxe + dxf \end{pmatrix}$

It is not possible to multiply BxA, since the number of columns of the first matrix is not equal to the number of rows of the second matrix

Example: 4

If B = $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and C = $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Find BC

BC = $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ = $\begin{pmatrix} 2x2 + 3x5 \\ 1x2 + 4x5 \end{pmatrix}$ = $\begin{pmatrix} 4 + 15 \\ 2 + 20 \end{pmatrix}$ = $\begin{pmatrix} 19 \\ 22 \end{pmatrix}$

Identity Matrix

Identity matrix is the square matrix whose elements in the leading diagonal are 1 and all other elements are zero and sometime is called unit matrix

I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

I = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Any matrix multiply by identity matrix is the same as the identity matrix multiply by the same matrix,

The singular matrix

The singular matrix is a square matrix whose determinant is 0. It is a matrix that does not have multiplicative inverse.

Example

A = $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

Determinant of A = $\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}$ = 4*1 -2*2 = 4 – 4 = 0

So if the determinant is 0 that matrix can't be solved

Chapter 2 Second Order Matrices

Second Order Matrices

Determinant of the second order matrix

Let A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Find determinant of A,

Det(A) = $|A|$ = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad – bc

Example: 5

If A = $\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}$

Find $|A|$

$|A|$ = $\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$ = 3x2 –(-4x1) = 6 + 4 = 10

Inverse of the second order matrix

Suppose A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

Inverse of A = A^-1 = $\frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ = $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Example: 6

Find the inverse of matrix A = $\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}$

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ = $\frac{1}{3x2-(-4x1)}\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ = $\frac{1}{6+4}\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ = $\begin{pmatrix} 2/10 & 4/10 \\ -1/10 & 3/10 \end{pmatrix}$

Example: 7

Solve 7x + 5y = 4 and 3x + 2y = 7 using matrix method

Solution:

7x + 5y = 4

3x + 2y = 7

Write in matrix form

$\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

Find inverse

$\frac{1}{\det(A)} \begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix} = \frac{1}{7x2-(5x3)} \begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix} = \frac{1}{14-15} \begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} \frac{2}{-1} & -5/-1 \\ -3/-1 & 7/-1 \end{pmatrix}$

$= \begin{pmatrix} \frac{2}{-1} & -5/-1 \\ -3/-1 & 7/-1 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}$

Multiply inverse to both sides

$\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

Matrix multiply by its inverse is equal to unit matrix,

$\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\begin{pmatrix} -2x7 + 3x5 & -2x5 + 5x2 \\ 3x7 + 3x - 7 & 5x3 + 2x - 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\begin{pmatrix} -14 + 15 & -10 + 10 \\ 21 - 21 & 15 - 14 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2 & 5\\3 & -7\end{pmatrix}\begin{pmatrix}4\\7\end{pmatrix} = \begin{pmatrix}-2x4 + 5x7\\3\ x4 + 7x - 7\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2x4 + 5x7\\3\ x4 + 7x - 7\end{pmatrix} = \begin{pmatrix}-8 + 35\\12 - 49\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-8 + 35\\12 - 49\end{pmatrix} = \begin{pmatrix}27\\-37\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = 1,$

Therefore $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}27\\-37\end{pmatrix}$

Hence the value of x = 27 and y = -37

Example: 8

Let the matrix A be given by A = $\begin{pmatrix}4 & 2\\3 & 2\end{pmatrix}$

(a) Find the inverse A^-1

(b) Find the values of x and y for which A$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}8\\7\end{pmatrix}$

Hence solve the equations:

4x + 2y = 16

3x + 2y = 14

Solution:

(a) Inverse of A

A = $\begin{pmatrix}4 & 2\\3 & 2\end{pmatrix}$

Determinant of A = (4x2 – 3x2) = 8-6 = 2

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}$ = $\frac{1}{2}\begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}$ = $\begin{pmatrix} 2/2 & -2/2 \\ -3/2 & 4/2 \end{pmatrix}$ = $\begin{pmatrix} 1 & -1 \\ -3/2 & 2 \end{pmatrix}$

Therefore, the inverse of A = $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}$

(b) Finding the values of x and y

$\begin{pmatrix} 4 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} 8 \\ 7 \end{pmatrix}$

In order to find the values of x and y we need to multiply by the inverse of A to both sides

$\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}\begin{pmatrix} 8 \\ 7 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 3 & 2 \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$,

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}\begin{pmatrix} 8 \\ 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-3/2} & \mathbf{2} \end{pmatrix}\begin{pmatrix} 8 \\ 7 \end{pmatrix}$ = $\begin{pmatrix} 8x1 + 7x - 1 \\ -\frac{3}{2}x8 + 7x2 \end{pmatrix}$ = $\begin{pmatrix} 8 - 7 \\ -12 + 14 \end{pmatrix}$ = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Therefore the values of x and y are 1 and 2 respectively

Example: 9

Use the inverse matrix to solve the equations:

3x + y = 8

2x + y = 3

Solution:

3x + y = 8

2x + y = 3

Write the equations in matrix form,

$\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ = (3x1 – 2x1) = 3-2 = 1

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} = \frac{1}{1}\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ 3 \end{pmatrix}$

But $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = 1$

$\begin{pmatrix}x\\y\end{pmatrix}$ = $\begin{pmatrix}1 & -1\\-2 & 3\end{pmatrix}\begin{pmatrix}8\\3\end{pmatrix}$ = $\begin{pmatrix}8x1 + 3x - 1\\8x - 2 + 3x3\end{pmatrix}$ = $\begin{pmatrix}8 - 3\\-16 + 9\end{pmatrix}$ = $\begin{pmatrix}5\\-7\end{pmatrix}$

Therefore the values of x and y are 5 and -7 respectively

Example: 10

Solve the following simultaneous equations by matrix method:

x + 4y = 8

x + 5y = 12

Solution:

Write the equations in matrix form,

$\begin{pmatrix}1 & 4\\1 & 5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ = $\begin{pmatrix}8\\12\end{pmatrix}$

Determinant of $\begin{pmatrix}1 & 4\\1 & 5\end{pmatrix}$ = (1x5 – 1x4) = 5-4 = 1

Inverse = $\frac{1}{\det(A)}\begin{pmatrix}5 & -4\\-1 & 1\end{pmatrix}$ = $\frac{1}{1}\begin{pmatrix}5 & -4\\-1 & 1\end{pmatrix}$ = $\begin{pmatrix}5 & -4\\-1 & 1\end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix}5 & -4\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & 4\\1 & 5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ = $\begin{pmatrix}5 & -4\\-1 & 1\end{pmatrix}\begin{pmatrix}8\\12\end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\\mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix} = \begin{pmatrix} 8x5 + 12x - 4 \\ 8x - 1 + 12x1 \end{pmatrix} = \begin{pmatrix} 40 - 48 \\ -8 + 12 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$

Therefore the values of x and y are -8 and 4 respectively

Example: 11

Solve by matrix method,

3x - 5y = 1

2x + 7y = 11

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix}$ = (3x7 – 2x-5) = 21 + 10 = 31

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix} = \frac{1}{31}\begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}\begin{pmatrix} 1 \\ 11 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}\begin{pmatrix} 1 \\ 11 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7/31 & 5/31 \\ -2/31 & 3/31 \end{pmatrix}\begin{pmatrix} 1 \\ 11 \end{pmatrix} = \begin{pmatrix} \frac{7}{31}x1 + \frac{5}{31}x11 \\ -\frac{2}{31}x1 + \frac{3}{31}x11 \end{pmatrix} = \begin{pmatrix} \frac{7}{31} + \frac{55}{31} \\ -\frac{2}{31} + \frac{33}{31} \end{pmatrix} = \begin{pmatrix} \frac{62}{31} \\ \frac{31}{31} \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Therefore the values of x and y are 2 and 1 respectively

Example: 12

2x + 3y = 340

4x + y = 280

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 340 \\ 280 \end{pmatrix}$

Determinant of $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$ = (2x1 – 4x3) = 2 - 12 = -10

Inverse = $\frac{1}{\det(A)}\begin{pmatrix}1 & -3\\ -4 & 2\end{pmatrix} = \frac{1}{-10}\begin{pmatrix}1 & -3\\ -4 & 2\end{pmatrix} = \begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$$\begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}\begin{pmatrix}2 & 3\\ 4 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}\begin{pmatrix}340\\ 280\end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}\begin{pmatrix}2 & 3\\ 4 & 1\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

$$\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}\begin{pmatrix}340\\ 280\end{pmatrix}$$

But $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 1$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{-1}{10} & \frac{3}{10}\\ \frac{4}{10} & \frac{-2}{10}\end{pmatrix}\begin{pmatrix}340\\ 280\end{pmatrix} = \begin{pmatrix}340x\frac{-1}{10} + 280x\frac{3}{10}\\ 340x\frac{4}{10} + 280x\frac{-2}{10}\end{pmatrix} = \begin{pmatrix}-34 + 28x3\\ 34x4 - 28x2\end{pmatrix} = \begin{pmatrix}50\\ 80\end{pmatrix}$$

Therefore the values of x and y are 50 and 80 respectively

Example: 13

Solve by matrix method,

x + y = 3

x – y = 7

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

Determinant of $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ = (1x-1 – 1x1) = -1 - 1 = -2

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x3 + \frac{1}{2}x7 \\ \frac{1}{2}x3 + -\frac{1}{2}x7 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x3 + \frac{1}{2}x7 \\ \frac{1}{2}x3 + -\frac{1}{2}x7 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} + \frac{7}{2} \\ \frac{3}{2} - \frac{7}{2} \end{pmatrix} = \begin{pmatrix} \frac{10}{2} \\ \frac{-4}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

Therefore the values of x and y are 5 and -2 respectively

Example: 14

Solve by matrix method,

2z + w = 3

3z + w = 9

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$

Determinant of $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ = (2x1 – 3x1) = 2-3 = -1

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} = \frac{1}{-1}\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$

The values of z and w,

Multiply both sides by the inverse

$\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 9 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} z \\ w \end{pmatrix}$ = $\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 9 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} z \\ w \end{pmatrix}$ = $\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ = $\begin{pmatrix} 3x - 1 + 9x1 \\ 3x3 + 9x - 2 \end{pmatrix}$ = $\begin{pmatrix} -3 + 9 \\ 9 - 18 \end{pmatrix}$ = $\begin{pmatrix} 6 \\ -9 \end{pmatrix}$

Therefore the values of z and w are 6 and -9 respectively

Example: 15

Solve by matrix method,

3x + 2y = 4

5x – 2y = 12

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & 2 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} 4 \\ 12 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & 2 \\ 5 & -2 \end{pmatrix}$ = (3x-2 – 5x2) = -6-10 = -16

Inverse = $\frac{1}{\det(A)}\begin{pmatrix}-2 & -2\\ -5 & 3\end{pmatrix} = \frac{1}{-16}\begin{pmatrix}-2 & -2\\ -5 & 3\end{pmatrix} = \begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$$\begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}\begin{pmatrix}3 & 2\\ 5 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}\begin{pmatrix}4\\ 12\end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}\begin{pmatrix}3 & 2\\ 5 & -2\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

$$\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}\begin{pmatrix}4\\ 12\end{pmatrix}$$

But $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 1$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{2}{16} & \frac{2}{16}\\ \frac{5}{16} & -\frac{3}{16}\end{pmatrix}\begin{pmatrix}4\\ 12\end{pmatrix} = \begin{pmatrix}\frac{2}{16}x4 + \frac{2}{16}x12\\ \frac{5}{16}x4 + \frac{-3}{16}x12\end{pmatrix} = \begin{pmatrix}\frac{8}{16} + \frac{24}{16}\\ \frac{20}{16} + \frac{-36}{16}\end{pmatrix} = \begin{pmatrix}\frac{32}{16}\\ \frac{-16}{16}\end{pmatrix} = \begin{pmatrix}2\\ -1\end{pmatrix}$$

Therefore the values of x and y are 2 and -1 respectively

Example: 16

Solve by matrix method,

3x + y = 7

3x – 4y = -13

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & 1 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -13 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & 1 \\ 3 & -4 \end{pmatrix}$ = (3x-4 – 3x1) = -12-3 = -15

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} -4 & -1 \\ -3 & 3 \end{pmatrix} = \frac{1}{-15}\begin{pmatrix} -4 & -1 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}\begin{pmatrix} 7 \\ -13 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}\begin{pmatrix} 7 \\ -13 \end{pmatrix}$$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{4}{15} & \frac{1}{15} \\ \frac{3}{15} & -\frac{3}{15} \end{pmatrix}\begin{pmatrix} 7 \\ -13 \end{pmatrix} = \begin{pmatrix} \frac{4}{15}x7 + \frac{1}{15}x - 13 \\ \frac{3}{15}x7 + \frac{-3}{15}x - 13 \end{pmatrix} = \begin{pmatrix} \frac{28}{15} - \frac{13}{15} \\ \frac{21}{16} + \frac{39}{16} \end{pmatrix} = \begin{pmatrix} \frac{15}{15} \\ \frac{60}{15} \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Therefore the values of x and y are 1 and 4 respectively

Example: 17

Solve by matrix method,

5q – 3p = 11

q + p = 7

Solution:

Write the equations in matrix form,

$$\begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix}$ = (5x1 – 1x-3) = 5+3= 8

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}$

The values of q and p,

Multiply both sides by the inverse

$$\begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}\begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}\begin{pmatrix} 11 \\ 7 \end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}\begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}\begin{pmatrix} 11 \\ 7 \end{pmatrix}$$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$$\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{-1}{8} & \frac{5}{8} \end{pmatrix}\begin{pmatrix} 11 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}x11 + \frac{3}{8}x7 \\ \frac{-1}{8}x11 + \frac{5}{8}x7 \end{pmatrix} = \begin{pmatrix} \frac{11}{8} + \frac{21}{8} \\ \frac{-11}{8} + \frac{35}{8} \end{pmatrix} = \begin{pmatrix} \frac{32}{8} \\ \frac{24}{8} \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Therefore the values of q and p are 4 and 3 respectively

Example: 18

Solve by matrix method,

7x + 5y = 4

3x + 2y = 7

Solution:

Write the equations in matrix form,

$$\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}$ = (7x2 – 3x5) = 14-15= -1

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix}$ = $\frac{1}{-1}\begin{pmatrix} 2 & -5 \\ -3 & 7 \end{pmatrix}$ = $\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 7 & 5 \\ 3 & 2 \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} -2 & 5 \\ 3 & -7 \end{pmatrix}\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ = $\begin{pmatrix} -2x4 + 7x5 \\ 4x3 + 7x - 7 \end{pmatrix}$ = $\begin{pmatrix} -8 + 35 \\ 12 - 49 \end{pmatrix}$ = $\begin{pmatrix} 27 \\ -37 \end{pmatrix}$

Therefore the values of x and y are 27 and -37 respectively

Example: 19

Solve by matrix method,

3z – 5w = 8

z + 3w = 12

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & -5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & -5 \\ 1 & 3 \end{pmatrix}$ = (3x3 – 1x-5) = 9+5= 14

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 3 & -5 \\ 1 & 3 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 3 & 5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}$

The values of z and w,

Multiply both sides by the inverse

$\begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$$\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} 3/14 & 5/14 \\ -1/14 & 3/14 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{3}{14}x8 + \frac{5}{14}x12 \\ -\frac{1}{14}x8 + \frac{3}{14}x12 \end{pmatrix} = \begin{pmatrix} \frac{24}{14} + \frac{60}{14} \\ -\frac{8}{14} + \frac{36}{14} \end{pmatrix} = \begin{pmatrix} \frac{84}{14} \\ \frac{-8+36}{14} \end{pmatrix} = \begin{pmatrix} \frac{84}{14} \\ \frac{28}{14} \end{pmatrix}$$

Therefore the values of z and w are 6 and 2 respectively

Example: 20

Solve by matrix method,

4q – 3p = 5

7q – 6p = 5

Solution:

Write the equations in matrix form,

$$\begin{pmatrix} 4 & -3 \\ 7 & -6 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 4 & -3 \\ 7 & -6 \end{pmatrix}$ = (4x-6 – 7x-3) = -24 + 21 = -3

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 4 & -3 \\ 7 & -6 \end{pmatrix} = \frac{1}{-3}\begin{pmatrix} -6 & 3 \\ -7 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix} =$

The values of q and p,

Multiply both sides by the inverse

$$\begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix}\begin{pmatrix} 4 & -3 \\ 7 & -6 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 7 & -6 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} -\frac{6}{-3} & \frac{3}{-3} \\ \frac{7}{-3} & \frac{4}{-3} \end{pmatrix} \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{-6}{-3}x5 + \frac{3}{-3}x5 \\ -\frac{7}{-3}x5 + \frac{4}{-3}x5 \end{pmatrix} = \begin{pmatrix} \frac{30}{3} - \frac{15}{3} \\ \frac{35}{3} - \frac{20}{3} \end{pmatrix} = \begin{pmatrix} \frac{15}{3} \\ \frac{15}{3} \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

Therefore the values of q and p are 5 and 5 respectively

Example: 21

Solve by matrix method,

3x + 2y = 14

x – 3y = 1

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 1 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & 2 \\ 1 & -3 \end{pmatrix}$ = (3x-3 – 1x2) = -9 -2 = -11

Inverse = $\frac{1}{\det(A)}\begin{pmatrix}3 & 2\\1 & -3\end{pmatrix} = \frac{1}{-11}\begin{pmatrix}-3 & -2\\-1 & 3\end{pmatrix} = \begin{pmatrix}\frac{-3}{-11} & \frac{-2}{-11}\\ \frac{-1}{-11} & \frac{3}{-11}\end{pmatrix} = \begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{-3}{11}\end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{3}{11}\end{pmatrix}\begin{pmatrix}3 & 2\\1 & -3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{3}{11}\end{pmatrix}\begin{pmatrix}14\\1\end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{3}{11}\end{pmatrix}\begin{pmatrix}3 & 2\\1 & -3\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

$\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{-3}{11}\end{pmatrix}\begin{pmatrix}14\\1\end{pmatrix}$

But $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = 1$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{3}{11} & \frac{2}{11}\\ \frac{1}{11} & \frac{-3}{11}\end{pmatrix}\begin{pmatrix}14\\1\end{pmatrix}$

$= \begin{pmatrix}\frac{3}{11}x14 + \frac{2}{11}x1\\ \frac{1}{11}\ x14 + \frac{-3}{11}x1\end{pmatrix} = \begin{pmatrix}\frac{3}{11}x14 + \frac{2}{11}x1\\ \frac{1}{11}\ x14 + \frac{-3}{11}x1\end{pmatrix} = \begin{pmatrix}\frac{42}{11} + \frac{2}{11}\\ \frac{14}{11} - \frac{3}{11}\end{pmatrix} = \begin{pmatrix}\frac{44}{11}\\ \frac{11}{11}\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}$

Therefore the values of x and y are 4 and 1 respectively

Example: 22

Solve by matrix method,

2x - 4y = 14

3x + y = 7

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \end{pmatrix}$

Determinant of $\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$ = (2x1 – 3x-4) = 2 +12 = +14

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}\begin{pmatrix} 14 \\ 7 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}\begin{pmatrix} 14 \\ 7 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{14} & \frac{4}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{pmatrix}\begin{pmatrix} 14 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{14}x14 + \frac{4}{14}x7 \\ \frac{-3}{14}\ x14 + \frac{2}{14}x7 \end{pmatrix} = \begin{pmatrix} \frac{14}{14} + \frac{4}{2} \\ \frac{-3}{1} + \frac{2}{2} \end{pmatrix} = \begin{pmatrix} 1+2 \\ -3+1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Therefore the values of x and y are 3 and -2 respectively

Example: 23

Solve by matrix method,

3x +2y = 20

-5x + 3y = 11

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 11 \end{pmatrix}$

Determinant of $\begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix}$ = (3x3 – 2x-5) = 9 +10 = +19

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 3 & -2 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$$\begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}\begin{pmatrix} 20 \\ 11 \end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}\begin{pmatrix} 20 \\ 11 \end{pmatrix}$$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{19} & \frac{-2}{19} \\ \frac{5}{19} & \frac{3}{19} \end{pmatrix}\begin{pmatrix} 20 \\ 11 \end{pmatrix} = \begin{pmatrix} \frac{3}{19}x20 + \frac{-2}{19}x11 \\ \frac{5}{19}\ x20 + \frac{3}{19}x11 \end{pmatrix} = \begin{pmatrix} \frac{60}{19} - \frac{22}{19} \\ \frac{100}{19} + \frac{33}{19} \end{pmatrix} = \begin{pmatrix} \frac{38}{19} \\ \frac{133}{19} \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

Therefore the values of x and y are 2 and 7 respectively

Example: 24

Solve by matrix method,

4s +5r = 1

3r – 2s = -17

Solution:

Rearrange the equations,

5r + 4s = 1

3r – 2s = -17

Write the equations in matrix form,

$\begin{pmatrix}5 & 4\\3 & -2\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}1\\-17\end{pmatrix}$

Determinant of $\begin{pmatrix}5 & 4\\3 & -2\end{pmatrix}$ = (5x-2 – 3x4) = -10 -12 = -22

Inverse = $\frac{1}{\det(A)}\begin{pmatrix}5 & 4\\3 & -2\end{pmatrix} = \frac{1}{-22}\begin{pmatrix}-2 & -4\\-3 & 5\end{pmatrix} = \begin{pmatrix}\frac{-2}{-22} & \frac{-4}{-22}\\ \frac{-3}{-22} & \frac{5}{-22}\end{pmatrix} = \begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}$

The values of r and s,

Multiply both sides by the inverse

$\begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}\begin{pmatrix}5 & 4\\3 & -2\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}\begin{pmatrix}1\\-17\end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}\begin{pmatrix}5 & 4\\3 & -2\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

$\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}\begin{pmatrix}1\\-17\end{pmatrix}$

But $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = 1$

$\begin{pmatrix}r\\s\end{pmatrix} = \begin{pmatrix}\frac{2}{22} & \frac{4}{22}\\ \frac{3}{22} & -\frac{5}{22}\end{pmatrix}\begin{pmatrix}1\\-17\end{pmatrix} = \begin{pmatrix}\frac{2}{22}x1 + \frac{4}{22}x - 17\\ \frac{3}{22}\ x1 + \frac{-5}{22}x - 17\end{pmatrix} = \begin{pmatrix}\frac{2}{22} - \frac{68}{22}\\ \frac{3}{22} + \frac{85}{22}\end{pmatrix} = \begin{pmatrix}-\frac{66}{22}\\ +\frac{88}{22}\end{pmatrix} = \begin{pmatrix}-3\\+4\end{pmatrix}$

Therefore the values of r and s are -3 and 4 respectively

Example: 25

Solve by matrix method,

2x +3y = 13

7x – 5y = -1

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 2 & 3 \\ 7 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \end{pmatrix}$

Determinant of $\begin{pmatrix} 2 & 3 \\ 7 & -5 \end{pmatrix}$ = (2x-5 – 7x3) = -10 -21 = -31

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 2 & 3 \\ 7 & -5 \end{pmatrix} = \frac{1}{-31}\begin{pmatrix} -5 & -3 \\ -7 & 2 \end{pmatrix} = \begin{pmatrix} \frac{-5}{-31} & \frac{-3}{-22} \\ \frac{-7}{-31} & \frac{2}{-31} \end{pmatrix} = \begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$\begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 7 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}\begin{pmatrix} 13 \\ -1 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 7 & -5 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}\begin{pmatrix} 13 \\ -1 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \frac{5}{31} & \frac{3}{31} \\ \frac{7}{31} & \frac{2}{-31} \end{pmatrix}\begin{pmatrix} 13 \\ -1 \end{pmatrix}$ = $\begin{pmatrix} \frac{5}{31}x13 + \frac{3}{31}x - 1 \\ \frac{7}{31}\ x13 + \frac{-2}{31}x - 1 \end{pmatrix} = \begin{pmatrix} \frac{65}{31} - \frac{3}{31} \\ \frac{91}{31} + \frac{2}{31} \end{pmatrix}$ = $\begin{pmatrix} \frac{62}{31} \\ \frac{93}{31} \end{pmatrix}$ = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Therefore the values of x and y are 2 and 3 respectively

Example: 26

Solve by matrix method,

6p -7q = -1

5p – 4q = 12

Solution:

Write the equations in matrix form,

$\begin{pmatrix} 6 & -7 \\ 5 & -4 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$ = $\begin{pmatrix} -1 \\ 12 \end{pmatrix}$

Determinant of $\begin{pmatrix} 6 & -7 \\ 5 & -4 \end{pmatrix}$ = (6x-4 – 5x-7) = -24 +35 = 11

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 6 & -7 \\ 5 & -4 \end{pmatrix}$ = $\frac{1}{11}\begin{pmatrix} -4 & 7 \\ -5 & 6 \end{pmatrix}$ = $\begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix}$

The values of p and q,

Multiply both sides by the inverse

$$\begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix} \begin{pmatrix} 6 & -7 \\ 5 & -4 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix} \begin{pmatrix} -1 \\ 12 \end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix} \begin{pmatrix} 6 & -7 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix} \begin{pmatrix} -1 \\ 12 \end{pmatrix}$$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{-4}{11} & \frac{7}{11} \\ \frac{-5}{11} & \frac{6}{11} \end{pmatrix} \begin{pmatrix} -1 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{-4}{11} x - 1 + \frac{7}{11} x12 \\ \frac{-5}{11} x - 1 + \frac{6}{11} x12 \end{pmatrix} = \begin{pmatrix} \frac{4}{11} + \frac{84}{11} \\ \frac{5}{11} + \frac{72}{11} \end{pmatrix} = \begin{pmatrix} \frac{88}{11} \\ \frac{77}{11} \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$$

Therefore the values of p and q are 8 and 7 respectively

Example: 27

Solve by matrix method,

x/6 + y/3 = 8

x/4 – y/9 = 1

Solution:

x/6 + y/3 = 8 …………………….(i)

x/4 – y/9 = 1……………………(ii)

Remove fraction by multiplying (i) by 6 to both sides and (ii) by 36 to both sides,

6 * x/6 + 6 * y/3 = 6 * 8

x + 2y = 48

and,

36 * x/4 – 36 * y/9 = 36 * 1

9x – 4y = 36

Now we have the new equations as follows,

x + 2y = 48

9x – 4y = 36

Write the equations in matrix form,

$\begin{pmatrix} 1 & 2 \\ 9 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 48 \\ 36 \end{pmatrix}$

Determinant of $\begin{pmatrix} 1 & 2 \\ 9 & -4 \end{pmatrix}$ = (1x-4 – 9x2) = -4 -18 = -22

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 1 & 2 \\ 9 & -4 \end{pmatrix} = \frac{1}{-22}\begin{pmatrix} -4 & -2 \\ -9 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-4}{-22} & \frac{-2}{-22} \\ \frac{-9}{-22} & \frac{1}{-22} \end{pmatrix} = \begin{pmatrix} \frac{4}{22} & \frac{2}{22} \\ \frac{9}{22} & \frac{1}{-22} \end{pmatrix}$

The values of x and y,

Multiply both sides by the inverse

$$\begin{pmatrix}\frac{4}{22} & \frac{2}{22}\\ \frac{9}{22} & \frac{1}{-22}\end{pmatrix}\begin{pmatrix}1 & 2\\ 9 & -4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{4}{22} & \frac{2}{22}\\ \frac{9}{22} & \frac{1}{-22}\end{pmatrix}\begin{pmatrix}48\\ 36\end{pmatrix}$$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

Therefore $\begin{pmatrix}\frac{4}{22} & \frac{2}{22}\\ \frac{9}{22} & \frac{1}{-22}\end{pmatrix}\begin{pmatrix}1 & 2\\ 9 & -4\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$

$$\begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{4}{22} & \frac{2}{22}\\ \frac{9}{22} & \frac{1}{-22}\end{pmatrix}\begin{pmatrix}48\\ 36\end{pmatrix}$$

But $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = 1$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}\frac{4}{22} & \frac{2}{22}\\ \frac{9}{22} & \frac{1}{-22}\end{pmatrix}\begin{pmatrix}48\\ 36\end{pmatrix} = \begin{pmatrix}\frac{4}{22}x48 + \frac{2}{22}x36\\ \frac{9}{22}\,x48 + \frac{1}{-22}x36\end{pmatrix} = \begin{pmatrix}\frac{192}{22} + \frac{72}{22}\\ \frac{432}{22} - \frac{36}{22}\end{pmatrix} = \begin{pmatrix}\frac{264}{22}\\ \frac{396}{22}\end{pmatrix} = \begin{pmatrix}12\\ 18\end{pmatrix}$$

Therefore the values of x and y are 12 and 18 respectively

Example: 28

Solve by matrix method,

m/6 + 2n/3 = 6

2n/5 – m/10 = 2

Solution:

m/6 + 2n/3 = 6 ……………………. (i)

2n/5 – m/10 = 2 ……………………(ii)

Remove fraction by multiplying (i) by 6 to both sides and (ii) by 10 to both sides,

6 * m/6 + 6 * 2n/3 = 6 * 6

m + 4n = 36

and,

10 * 2n/5 – 10 * m/10 = 10 * 2

4n – m = 20

Now we have the new equations as follows,

m + 4n = 36

4n – m = 20

Rearrange the equations to align the variables, all equations to start with m

m + 4n = 36

– m + 4n = 20

Write the equations in matrix form,

$$\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 36 \\ 20 \end{pmatrix}$$

Determinant of $\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}$ = (1x4 – 4x-1) = 4 +4 = 8

Inverse = $\frac{1}{\det(A)}\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}$ = $\frac{1}{8}\begin{pmatrix} 4 & -4 \\ 1 & 1 \end{pmatrix}$ = $\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

The values of m and n,

Multiply both sides by the inverse

$\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix}$ = $\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 36 \\ 20 \end{pmatrix}$

Matrix multiplies by its inverse the value is unit matrix $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Therefore $\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}$ = $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ = $\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 36 \\ 20 \end{pmatrix}$

But $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\begin{pmatrix} m \\ n \end{pmatrix}$ = $\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{8} & \frac{1}{8} \end{pmatrix}\begin{pmatrix} 36 \\ 20 \end{pmatrix}$ = $\begin{pmatrix} \frac{1}{2}x36 + \frac{-1}{2}x20 \\ \frac{1}{8}\,x36 + \frac{1}{8}x20 \end{pmatrix}$ = $\begin{pmatrix} 18 - 10 \\ \frac{9}{2} + \frac{15}{2} \end{pmatrix}$ = $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

Therefore the values of m and n are 8 and 12 respectively

Chapter 3 Third Order Matrices

Third Order Matrices

1: Determinant

A = $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}$

Determinant A = $\begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix}$

$$= a11\begin{vmatrix} a22 & a23 \\ a32 & a33 \end{vmatrix} - a12\begin{vmatrix} a21 & a23 \\ a31 & a33 \end{vmatrix} + a13\begin{vmatrix} a21 & a22 \\ a31 & a32 \end{vmatrix}$$

$$= a11(a22a33 - a23a32) - a12(a21a33 - a23a31) + a13(a21a32 - a22a31)$$

Example:

Let A = $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$; Find | A |

Solution:

| A | = $\begin{vmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{vmatrix}$

$$= 1\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix}$$

= 1[(2x2) – (5x-1)] + 2[(0x2) – (-4x-1] + 3[(0x5 –(-4x2)]

= 1(4+5) + 2(0-4) +3(0+8)

= 9 – 8 + 24

= 25

Therefore determinant of matrix A is 25

2: Minor

Minor of any entry is the determinant obtained by deleting the row and column in which the entry stand.

Example :

A = $\begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix}$

The Minor of **a_{11}** = $\begin{vmatrix} \boldsymbol{a22} & \boldsymbol{a23} \\ \boldsymbol{a32} & \boldsymbol{a33} \end{vmatrix}$

The Minor of **a_{12}** = $\begin{vmatrix} \boldsymbol{a21} & \boldsymbol{a23} \\ \boldsymbol{a31} & \boldsymbol{a33} \end{vmatrix}$

The Minor of **a_{13}** = $\begin{vmatrix} \boldsymbol{a21} & \boldsymbol{a22} \\ \boldsymbol{a31} & \boldsymbol{a32} \end{vmatrix}$

The Minor of **a_{21}** = $\begin{vmatrix} \boldsymbol{a12} & \boldsymbol{a13} \\ \boldsymbol{a32} & \boldsymbol{a33} \end{vmatrix}$

The Minor of **a_{22}** = $\begin{vmatrix} \boldsymbol{a11} & \boldsymbol{a13} \\ \boldsymbol{a31} & \boldsymbol{a33} \end{vmatrix}$

The Minor of a_{23} = $\begin{vmatrix} a11 & a12 \\ a31 & a32 \end{vmatrix}$

The Minor of a_{31} = $\begin{vmatrix} a12 & a13 \\ a22 & a23 \end{vmatrix}$

The Minor of a_{32} = $\begin{vmatrix} a11 & a13 \\ a21 & a23 \end{vmatrix}$

The Minor of a_{33} = $\begin{vmatrix} a11 & a12 \\ a21 & a22 \end{vmatrix}$

3: Cofactor

The cofactor of an entry is defined to be the minor of the entry together with its sign

i.e **Cofactor a_{ij} = c_{ij} = {(-)^i+j}Mij**

where i = the first number of an element

j = the second number of an element

Example:

Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$

i) Minor of a11 = $\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix}$

= 2*2 –(5*-1)

= 4 + 5

= 9

And cofactor of a_{11} = [(-)^1+1] * 9

=[(-1)^2]*9

= 1*9

= 9

ii) Minor of a_{12} = $\begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix}$

= 0*2 – (-4*-1)

= 0 – 4

=-4

And its Cofactor = [(-1)^1+2]* -4

= $(-1)^3$ * -4

= -1 * -4

= 4

iii) Minor of a_{13} = $\begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix}$

= 0*5 – (-4*2)

= 0 + 8

=8

And its Cofactor = [(-1)^1+3]* 8

= $(-1)^3$ * 8

= -1 * 8

= -8

iv) Minor of a_{21} = $\begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix}$

= -2*2 – 5*3

= -4 – 15

= -19

And its Cofactor = [(-1)^2+1]* -19

= [(-1)^3] * -19

= -1 * -19

= 19

v) Minor of a_{22} = $\begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix}$

= 1*2 – (-4*3)

= 2 +12

= 14

And its Cofactor = [(-1)^2+2]* 14

= [(-1)^4] * 14

= 1 * 14

= 14

vi) Minor of a_{23} = $\begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix}$

= 1*5 – (-4*-2)

= 5 - 8

= -3

And its Cofactor = [(-1)^2+3]* -3

= [(-1)^5] * -3

= -1 * -3

= 3

vii) Minor of a_{31} = $\begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix}$

= -2*-1 – (2*3)

= 2 - 6

= -4

And its Cofactor = [(-1)^3+1]* -4

= [(-1)^4] * -4

= 1 * -4

= -4

vii) Minor of a_{32} = $\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix}$

= 1*-1 – (0*3)

= -1-0

= -1

And its Cofactor = [(-1)^3+2]* -1

= [(-1)^4] * -1

= 1 * -1

= -1

ix) Minor of a_{33} = $\begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix}$

= 1*2 – (0*-2)

= 2 - 0

= 2

And its Cofactor = [(-1)^3+3]* 2

= [(-1)^6] * -1

= 1 * -1

= -1

Therefore Cofactor of A = $\begin{pmatrix} 9 & 4 & -8 \\ 19 & 14 & 3 \\ -4 & -1 & -1 \end{pmatrix}$

4: Adjoint of a matrix

The adjoint of a matrix A is defined to be the transpose of matrix obtained from A by replacing each entry by its cofactor

It is denoted by Adj A

Adj A = [Aij]^T

If A = $\begin{pmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{pmatrix}$

And cofactors of entries: -

= $\begin{pmatrix} A1 & B1 & C1 \\ A2 & B2 & C2 \\ A3 & B3 & C3 \end{pmatrix}$

Adj A = $\begin{pmatrix} A1 & A2 & A3 \\ B1 & B2 & B3 \\ C1 & C2 & C3 \end{pmatrix}$

Example if A = $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$

Cofactor of A = $\begin{pmatrix} 9 & 4 & -8 \\ 19 & 14 & 3 \\ -4 & -1 & -1 \end{pmatrix}$ from above example

And

Adjoint of A = $\begin{pmatrix} 9 & 19 & -4 \\ 4 & 14 & -1 \\ -8 & 3 & -1 \end{pmatrix}$

5: Inverse of a matrix

Only for non – singular or invertible matrices

Inverse = A^-1 = $\frac{1}{|A|}${ Adj A}

Example:

Solve by matrices (inverse method)

x + y + z = 3

x + 2y + 3z = 4

x + 4y + 9z = 6

Solution:

Matrix form,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

Determinant $= 1\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$

= 1(18 -12) -1(9-3) +1(4-2)

= 6 – 6 +2

= 2

Cofactor:

First row = $1\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$

= 1(18 -12) -1(9-3) +1(4-2)

= 1(6) -1(6) +1(2), the numbers in brackets are minors, minors multiply with the sign belongs to the particular entry you get the cofactor

$= 6 \quad -6 \quad 2$

Second row = $1\begin{vmatrix}1 & 1\\4 & 9\end{vmatrix}$ - $2\begin{vmatrix}1 & 1\\1 & 9\end{vmatrix}$ + $3\begin{vmatrix}1 & 1\\1 & 4\end{vmatrix}$

= 1(9 -4) -1(9-1) +1(4-1)

= 1(5) -1(8) +1(3), the numbers in brackets are minors, minors multiply with the sign belongs to the particular entry you get the cofactor

$= -5 \quad 8 \quad -3$

Third row = $1\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}$ - $4\begin{vmatrix}1 & 1\\1 & 3\end{vmatrix}$ + $9\begin{vmatrix}1 & 1\\1 & 2\end{vmatrix}$

= 1(3 -2) -1(3-1) +1(2-1)

= 1(1) -1(2) +1(1), the numbers in brackets are minors, minors multiply with the sign belongs to the particular entry you get the cofactor

$= 1 \quad -2 \quad 1$

Therefore the Cofactors = $\begin{pmatrix}6 & -6 & 2\\-5 & 8 & -3\\1 & -2 & 1\end{pmatrix}$

Adjoint of Matrix is obtained by Transposition whereby :

The first row of cofactor will be the first column of the Adjoint

The second row of cofactor will be the second column of the Adjoint

The third row of cofactor will be the third column of the Adjoint

Adjoint = C^T = $\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$ =

A^-1 = $\frac{Adjoint}{|A|}$

$= \frac{1}{2}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$

Therefore,

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 18 - 20 + 6 \\ -18 + 32 - 12 \\ 6 - 12 + 6 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

Therefore x = 2, y = 1 and z = 0

Question: Solve the following system of simultaneous equations by substitution method;

$3y + 2x = z + 1$

$3x + 2z = 8 – 5y$

$3z – 1 = x – 2y$

Solution:

$3y + 2x = z + 1$ ……………….(i)

$3x + 2z = 8 – 5y$ ……………..(ii)

$3z – 1 = x – 2y$………………..(iii)

From (i), $z = 3y + 2x – 1$…………(iv)

Substitute (iv) into (ii) and (iii)

$3x + 2(3y + 2x – 1) = 8 – 5y$

$3x + 6y + 4x – 2 - 8 + 5y = 0$

$7x + 11y -10 = 0$

$7x + 11y = 10$ ………………………(v)

$3(3y + 2x – 1) – 1 = x – 2y$

$9y + 6x – 3 – 1 = x – 2y$

$9y + 6x – 3 – 1 – x + 2y = 0$

$9y + 2y + 6x – x – 4 = 0$

$11y + 5x = 4$

$5x + 11y = 4$ …………………..(vi)

Subtract (vi) from (v)

7x – 5x = 10 – 4

2x = 6, giving x = 3

Substitute the value of x = 3 into (v)

7*3 + 11y = 10

21 + 11y = 10

11y = 10 – 21

11y = -11 giving y = -1

Substitute the value of x and y into (i)

3y + 2x = z + 1

3*-1 + 2*3 = z + 1

-3 + 6 = z + 1

3 -1 = z

2 = z

Hence the value of x = 3, y = -1 and z = 2

SOLVING MATRIX BY USING CRAMMER'S METHOD(CRAMMER'S RULE)

Example: Solve by Crammer's method

x + y + z = 3

x + 2y + 3z = 4

x + 4y + 9z = 6

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

Determinant = 2 as before

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 3 \\ 6 & 4 & 9 \end{vmatrix}}{determinant}$$

$= \{3\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} -1\begin{vmatrix} 4 & 3 \\ 6 & 9 \end{vmatrix} +1\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}\}/\text{determinant}=2$

= (18 – 18 +4)/2

= 2

$$y = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 4 & 3 \\ 1 & 6 & 9 \end{vmatrix}}{determinant}$$

$= \{1\begin{vmatrix} 4 & 3 \\ 6 & 9 \end{vmatrix} -3\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} +1\begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix}\}/\text{determinant}=2$

= (18 – 18 +2)/2

= 1

$$z = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{vmatrix}}{determinant}$$

$= \{1\begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix} -1\begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix} +3\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}\}/\text{determinant}=2$

= (-4 – 2 +6)/2

= 0

Therefore x = 2, y = 1 and z = 0 as before

Resources

Loyal Academy is the education platform for Q&A for school subjects from Primary School to University.

Contact Information:

- Website: https://loyalacademy.co.tz
- E-mail: ndege66@gmail.com
- Social media:

  Facebook: Loyal Academy Tanzania
- Phone number: 0754 565136